

education, for instance, designing online courses, where one of the key challenges is a need for expensive laboratory equipment. The systems for modeling electronic circuits and robotic engineering have been considered. The performed analysis is based on practical use of virtual laboratories in the

course developed by the authors "Bases of practical engineering modeling" for pupils. It is shown that use of virtual laboratories allows pupils to learn the issues of robotic engineering system design without special equipment.

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## Rewarding Learning of Maths in Engineering Schools: Laboratory Works

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The approach to development and use of laboratory works in training discrete mathematics and mathematical logic is suggested in the article. It is based on the computer tools to develop and improve productive thinking. The works involved are based on modeling subject field, they include target setting which determines students experimental and constructive activities as well as resources for automatical evaluation of partial solutions submitted by students. Experiment results have shown a significant increase in efficiency as compared to the multiple choice tests.

**Key words:** laboratory works, discrete mathematics, mathematical logic

#### Introduction

A laboratory work on discrete mathematics is often defined by most of contemporary authors as a set of problems on a definite subject aimed to develop students' certain skills. It seems to us that under such an approach, performance of laboratory work is no different from solution of individual home task or a set of tests [1-3].

Traditional engineering education interprets laboratory work on mathematics as a training session being a basic unit of laboratory (calculating) practice, using numerical techniques to solve professional problems [4]. However, the solution of routine problems relevant to their future job is a relatively rare case, they are mostly "professional" engineering problems often called the applied problems.

Laboratory practice is to consist of examples of problem solutions modelled on the basis of creative individual tasks. In this case laboratory tasks intensify students' independent work, contribute to better understanding of the subject and master their problem solution skills. In the classroom students take an active part in

solution and analysis of the problems that they have to solve individually. Independent students' solution of the problems promotes their better understanding of the theory and develops their practical skills of managing the tasks that relate to studying the discipline "Discrete mathematics".

The training process will become more efficient if its participants solve non-trivial substantial problems and, for this purpose, they need to adopt new methods and tools including corresponding theory. Besides, the idea of learning process as a kind of research work increases the students' motivation.

Hence, laboratory work on discrete mathematics is a basis for acquiring research competency of future engineers.

It should be noted that the terms "laboratory work" and "laboratory practice" are often used in the literature as synonyms. These terms are better to be distinguished. The basic difference of laboratory practice from a laboratory work consists in systematic nature of the former. The practice includes several laboratory works different in subject and, sufficiently isolated in time of performance, but united by common goal relative to a student's specific training area.



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To perform laboratory work of research type a number of scholars suggest active use of application packages (Maple, Mathematica, MatLAB etc.). Making solution of complex problems easier, they help students to overcome psychological barrier in learning mathematics and make the training process more interesting and less complicated. In this case students are able to solve of more complex problems compensating for the lack of their knowledge by using package capacity, learning to present the research results in the form of concise substantial reports.

As an example one can give the laboratory practice designed at P.G. Demidov Yaroslavl State University [5]. The practice was developed using Mathematical package and covers such parts of discrete mathematics course as "Theory of combinations", "Graph algorithms", "Boolean functions", "Alphabetic coding", and "k-valued functions".

It should be noted that there is a risk of turning computer laboratory work into calculation of some task using template, ignoring the research constituent. Computerization of calculation did not only solve this problem, but even worsen it shifting from the result analysis to the capacity of software [6].

Of significant interest is the trend to use Moodle virtual learning environment to introduce laboratory practices into learning process.

In [7] the authors have presented the review of laboratory practice developed at Moscow Institute of Physics and Technology. The laboratory practice was on the course "Fuzzy models of discrete mathematics" and integrated in the training process. Computerized practices provided visual support of the learning material, a variety of tasks, monitoring performed works, and detection of mistakes. The practice was developed on AdobeFlash platform. To introduce the practice into training, Moodle virtual learning environment is used to integrate the developed live content by means of the SCORM standard.

At present, to develop research problems on Graph Theory, Maple package is widely used to give good visualization, which is of no small importance for this course. The teaching materials on "Discrete mathematics" designed at National Research Moscow Power Engineering Institute based on the textbook may serve as a bright example [8]. The materials cover most of topics of graph theory included in the curriculum: search for minimal spanning tree, the shortest paths in graph, network saturation, coverings, colorings, Hungarian algorithm, Hamiltonian and Eulerian cycles, encoding trees, etc.

Let us give some more examples of successful cases of learning materials including laboratory works with research tasks.

In [9] the authors suggest a number of laboratory works on "Discrete mathematics" course intended for learning systems of computer mathematics and related software. For example, it includes study of Combination Theory section using Mathematica package, automaton modeling in Electronics Workbench environment and solution of fuzzy logic problems in Fuzzy Logic package of MathLab environment.

The workers of Siberian Federal University have developed a set of laboratory works [10] including a number of learning syllabuses on the Graph Theory. This set is also interesting by the fact that it has a build-up universal unit of computer testing Unitest developed by the university workers and intended for intermediate and final monitoring.

In Yaroslavl-the-Wise Novgorod State University [11] a set of numerical problems has been suggested using the concept of logic programming. Of particular interest are the problems of human intellectual activity simulation in processing various types of information. The laboratory works also allow acquiring skills of writing and checkout of programmes in Prolog language to build up intellectual systems for different purposes.

As a conclusion of reviewing the computerized laboratory complexes the

latest development by Vyatka State University [12] should be considered. A set of emulators is suggested to study complicated sections of discrete mathematics. The emulators show the methods of solving such problems as: set operations, power of cluster sets, network and graph problem solutions, etc.

In addition to "computerized" laboratory works, mention should be made that the themes of course papers on discrete mathematics suggested by different authors [13] can also be used as creative laboratory tasks. Such course papers of small volume can be employed as topics for several laboratory works on corresponding theme.

In works [14-19] there is an approach to learning mathematics based on interaction with subject models of the concepts and evaluation of partial definition of those notions. This approach makes possible for students to have computer support of their cognitive activity. It is also interesting to consider using integration of two different interpretations of studied concepts. In this case when writing a problem a problem description is used as a set of its solution conditions, whereas in the process of solution – another, for example, algorithmic description of solution is used [20, 21].

#### Psychological and methodological bases of using computer tools in education

The approach to laboratory works on discrete mathematics, mathematical logics, and algorithm theory described in the article is based on computer tools to support productive thinking [22, 23].

The approach is based on Wertheimer's works, where a concept of productive thinking was first introduced [24], Vygotsky's research on the role of tools in human mind development [25], Leontiev's works on mechanism of interiorization [26] and those by Yakimanskaya concerned with using this mechanism to form significant mathematical notions [27], Krylov's research on methodical aspects of engineering education [28], Kudryavtsev's works on the structure of engineering thinking [29], Papert's study dealing with the role of tools

in the formation of mathematical concepts [30], Polya's works on research approach in studying mathematics [31, 32], research on the theory of IT environment [22] and information space [33].

Wertheimer [24] has shown that development of mathematical concepts is possible only with the support on basic concepts which are used by the students. Another related effect is understanding, which is a student's subjective feeling.

Use of analogues and models is connected with applied focus of teaching that is well formulated by academician A. N. Krylov [28]: "...any engineer should be a practioner, technician, ... is to develop not only his mind, but also his feelings so that they would not deceive him, he has not only to be able to look, but also see ... has to reduce his considerations not to Descartes' humble conclusion "I think, therefore, I am", but to a firm practical idea "I see, hear, touch, feel, therefore, it is so".

Kudryavtsev [29] has introduced a concept of engineering thinking. The main feature of engineering knowledge is that it is conceptual – visual – actual knowledge. In other words, dealing with a definite object it is necessary to have its structural image in the mind and perform some specific actions depending on this structure.

#### Modeling mathematical concepts and using computer models to teach mathematical logics and algorithm theory

As mentioned above, one of the ways to form a concept in a student's mind is to put the concept outside, substantiate it, make possible to handle it as an external object.

The challenge consists in the fact that, initially, the intellect structure of any student is unknown, so, the means to perform individual training are still absent. Therefore, an important task is to reveal and use common methods that initiate each student's activity in acquiring new ideas. It requires revising the basic concepts and using information environment to model something that will turn into every student's internal intellectual structure in the process of interiorization.

For this purpose, materialized simulated model should be simple enough, thus, to provide its flexibility and allow every student to use his/her own internal interpretation – *model should give intellectual freedom to students* [19]. By no means, it cannot be based on rigid teaching methods that make students, directly or indirectly, adjust to it. Vice versa, working with the model, a student projects it to his/her intuitive ideas that can contradict substantial aspects of a concept securely fixed by the model, and create a problem-based situation leading to new knowledge formation

Let us enumerate the basic properties of the models underlying laboratory models: the first is *flexibility of interpretation*, the second is *setup stiffness of substantial properties*. It should be also noted that the objects with these properties are called “boundary objects” in the theory of information environment [33], by means of which communities exchange information via information space.

Laboratory work will mean a set of the following elements:

- model of subject area related to the concept under study;
- target setting defining students’ experimental-constructive activity;
- tools of automatic evaluation of partial solutions submitted by students.

At first glance, it seems that if we have a formal definition of a concept, the model of subject area is built automatically. However, it is far from being so, as user interface is of great significance, which has to limit student actions to the extent that it should provide freedom in generating different solutions, but in a strictly limited area.

Let us give an example of a model. The problems of building finite-state automation are placed in the environment for graph constructing of finite-state machine. One of the methodical problems that should be solved in designing interface is to use a model of non-deterministic or deterministic machine. The choice of deterministic machine was defined by the problem essence. However, one can use the model of non-deterministic machine and check a determinacy as a part

of problem. It is these solutions that should be performed by an educator in terms of freedom degree given to a student [19]. As mentioned above, it should be sufficient to implement his/her own ideas and not to direct the students’ activity towards the predetermined solution. On the other hand, if a model has too many free parameters, a student has a possibility to experiment with parameters that the educator has connected with another problem. Hence, in the given example the model does not allow a student to build non-deterministic machine. It can be considered as a feedback contributing to better students’ understanding of a set problem.

The limitation of model operating environment is not the only way to obtain the feedback. The key element of feedback is the possibility to check the experimentally-based solutions using examples.

For example, a designed identification machine can be verified by different input symbol string. It makes possible to react to a particular solution. To check the final solution, the algorithm of equivalency checking is used.

Thus, we are able to formulate the idea: *when developing supporting software of learning mathematics it is necessary to transfer pedagogical techniques, the most appropriate for the set targets, to the new environment. The goal of this transfer is to automate monitoring of students’ productive activity providing them with cognitive freedom consistent with the pedagogical task.*

#### Experiment and Initial Results

Based on the presented approach four laboratory works were developed, each of which included four problems of various complexities. The themes of works were “Logic circuits”, “Finite-state automation and regular expressions”, “Predicates and quantifiers (Tarski’s world)”, “Turing machine”. 3 student classes participated in the experiment – 20 student groups, about 300 students. The experiment outcomes were estimated in three ways: examination results, teacher evaluation, and students’ questionnaire.

The number of problems solved correctly at a written test grew by 20%, whereas the mistakes in problem solution of laboratory works were twice as little as compared to solving the problems without the support of laboratory work. Based on the students’

answers, it can be concluded that the effect of laboratory work on mastering the concepts is sufficiently higher than the effect of traditional multiple choice tests that was performed simultaneously with laboratory works.

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